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APPLIED MATHEMATICS.

GEORGE W. MYERS.

SYLLABUS FOR DISCUSSION IN FACULTY CONFERENCE.

SCOPE OF MATHEMATICAL WORK IN THE SEVERAL GRADES OF THE SCHOOL OF EDUCATION.

THE teaching of number in the School of Education is based upon the following three principles:

I. The concept of number is best learned by generalizing from the concrete.

II. Number is an instrument through which the mind seeks to master the physical world.

III. The aim of teaching is not *instruction*, but *education*.

FIRST GRADE.

The mental acts in measuring and numbering are so similar and so dependent upon each other as to make it quite impossible to pronounce upon the psychological precedence of either. Real measurement involves the idea of number, and rational numbering implies measurement. But since numbering is purely a mental act, while the mental acts in measurement have their physical counterparts, measurement is much more readily accessible to the child than is the mental act of numbering. It is, therefore, obvious that measurement furnishes the easiest avenue of approach to the abstract concept of number. Accordingly, in the first and second grades the number work begins by the measurement of such things as the pupil can handle, and such as need to be measured for *other than pure number purposes.*

As an abstract outcome of the number work of the first grade the children are expected to know—

(1) The whole numbers and their notation up to 12 or 15. (2) The meaning of +, read "and" at first, later "plus;" –, read at first "less" or "taken from," later "minus;" ×, read "times" or "multiplied by;" ÷, read at first "is how many times," later "divided by;" and =, read at first "are," later "equals." (3) All the fundamental addition facts, such as $2+3=5$, $4+5=9$, etc., using sums not exceeding 12 or 15. (4) All the correlative subtraction

facts, such as 5−2=3, 5−3=2, 9−4=5, 9−5=4; etc. (5) Halves, thirds, fourths, and fifths of numbers up to about 15, and how to write and read them. (6) Yard, foot, inch, pint, quart, peck, bushel, gallon, cent, nickel, dime, and dollar, and their abbreviations, learned by using the measures.

SECOND GRADE.

The second stage on the way leading most advantageously to a clear and cogent idea of number is that in which the pupil works with numbers which have to do with concrete things, the things being absent from the senses. This occupies a prominent place in the second-grade number work. The first two stages of the work having shown the necessity of a working familiarity with the number processes used thus far, sufficient drill work, drawn largely from the other subjects, is given to secure this end.

At the end of the second year the pupils are expected to know—

(1) Numbers, their notation and use, so far as is necessary for a full working comprehension of two-figure numbers. (2) Meaning of the "place" of a figure and the value of a digit as dependent upon its place. (3) Addition and subtraction facts applied to one-figure numbers automatically, and to a considerable range of two-figure numbers. (4) Multiplication and division facts implicit in the products of the multiplication table as far as the fives, inclusive. (5) All of the standard units of measure learned during the first year, as well as the rest of the units upon which the most important tables are based. The units of the metric system must be included. The tables are to be learned through use. (6) Meaning of the terms *addition*, *add*, *sum*, *subtraction*, *subtract*, *remainder*, and *difference*, secured through working with the ideas. (7) After children have learned the facts of (3) and (4) through the use of intrinsically useful things, free use is made of any available objects for drill work to such an extent as to render these processes sufficiently automatic to be of real practical value. Like all instruments for the accomplishment of human purposes, number is about as likely to be harmful as helpful, unless some degree of skill in using it is attained. So far from extending the grasp of the mental fingers, the mere manipulation of the tool may even cripple the fingers; and the sharper the tool, the greater is the danger. So long as the pupil is struggling to learn the tool and its use, it would be absurd to expect him to use it effectively. It is the skill born of interested practice which makes the successful baseball catcher. And so the aim of the teacher should be, not to eliminate drill in number-teaching, but rather to make it interesting.

THIRD GRADE.

Proceeding on the idea that sound number-teaching should start with objects requiring measurement, then pass to and through the consideration of numbers relating to things (the things not being before the senses), then to abstract numbers, and finally back to things again by means of a wide range and variety of applications to practical problems, is the rational order of steps. In entering upon the third stage the necessities for abstraction and generalization must receive some attention; and this is given considerable place in the third year. While the great danger in number-teaching is in overdoing *routine* drill in the earlier stages, it must not be forgotten that there is also danger of grave and unsalutary consequences from failure to give *enough* drill of the *right kind.*

The work of the third year is expected to accomplish a knowledge—

(1) Of notation and numeration of numbers of two and three digits; (2) of number facts of the multiplication table to the twelves, together with their division correlates, until they become intuitive; (3) the meaning of digit, factor, multiplication, multiplicand, multiplier, product, division, dividend, divisor, quotient, square, cube, and period (a group of three figures); (4) of all the tables perfectly through using them in measurement; (5) the metric system, mastered through use; (6) of many exercises in geometrical drawing and construction.

FOURTH GRADE.

During the fourth grade chief emphasis rests upon the teaching of common and decimal fractions, though much has been done in these subjects before this grade is reached. Enough of the preceding and succeeding arithmetical subjects must be taken up in the class-room to bring out clearly the organic relation of fractions to other topics.

The following requirements are to be met by the end of the fourth school year:

(1) The meaning of the following terms: fraction; common, proper, improper, complex, decimal fractions; numerator, denominator, terms of a fraction, multiple, common multiple, least common multiple, common divisor, least common divisor, and reciprocal of a fraction. (2) The principles in accordance with which the four fundamental arithmetical operations are to

be applied to common and decimal fractions, the pupils being guided to the discovery of the rules for applying these principles. *Rules must not be neglected, but rationally taught.* Similarly for the simplifying and reducing of fractions. (3) The least common multiple and greatest common divisor learned as a means to the necessary reductions of fractions. (4) An extended knowledge of the arithmetical equation and its uses, and also some practice in the handling of simple algebraic equations. (5) Much practice with simple algebraic exercises introduced in closest connection with the arithmetical exercises to which they are analogous. (6) Some training in geometrical drawing and designing. (7) Field work, such as ranging out, locating, and measuring lines, angles, and areas called for in nature study and geography.

FIFTH GRADE.

By the end of the fifth grade the pupil is expected to have gained—

(1) An easy mastery of the four fundamental operations as applied to fractions of all kinds, and that he has put into the form of algebraic equations many of the principles underlying the treatment of fractions. (2) Through proper usage, a correct terminology, which may be used henceforth in all mathematical work. (3) A fund of ideas relating to algebraic number and to the equation. (4) A practical knowledge of some fundamental principles of similarity and of their possible uses in surveying; also an acquaintance with many important geometrical figures met in field work. (5) The organic connection of percentage with common fractions; also the economic value of a knowledge of percentage, through nature-study problems, etc. (6) Training of the judgment through a critical examination into the relative merits of different ways of handling the same problems. (7) Through the indirect measurements demanded by field work, a feeling of the real need of the more advanced methods of geometry and algebra.

SIXTH GRADE.

By the close of the sixth school year the pupil is expected to have—

(1) A practical mastery of common and decimal fractions, simple, complex, and compound—so far as this has not been secured in the fifth grade; (2) a mastery of percentage through problems drawn largely from nature study, and of simple interest through the computation and use of interest tables; (3) an introduction to elementary topographic surveying, and considerable knowledge of how to represent objects to scale; (4) some first-hand knowledge of the relative advantages of algebraic over arithmetical methods of treating problems; (5) a sufficient knowledge of pure arithmetic for all practical purposes, and to serve as a sound basis for elementary

geometry and algebra; (6) a degree of facility in attacking a concrete problem arithmetically; (7) some power to analyze a concrete problem into its abstract elements.

SEVENTH GRADE.

During the seventh school year chief emphasis rests upon the transition from arithmetic to algebra and geometry. The arithmetic does not stop, but its main uses here are to throw light on the first steps into the more advanced subjects. Inference and generalization from arithmetical processes and principles are the central activity about which the year's work turns. The three subjects are carried along in about equal proportions for a time, the work in arithmetic consisting of a re-survey, as advance work requires, of whatever has preceded in the first six grades; the algebra consisting in the main of extensions of the principles of arithmetic, and the geometry of representations to the eye of the algebraic and arithmetical truths. Later the algebra and geometry assume greater and greater prominence relatively, and by the end of the year the work is practically elementary algebra and geometry.

This grade is expected to accomplish the following:

(1) A clear perception that arithmetic, algebra, and geometry are not essentially different subjects. The difference, so far as it exists, consists merely of extensions of the ideas and principles of arithmetic. (2) That some things can be done more easily by arithmetic, some by geometry, and some by algebra. (3) A knowledge of the methods of root extraction arithmetically, geometrically, and algebraically. (4) A knowledge of the laws of transformation and use of the simple equation in one or two unknowns. (5) A re-examination of arithmetical truths in the light of algebra and geometry, (6) Graphical representation of scientific and statistical data, together with the interpretation of the curves. (7) Continual use, as occasion requires, of all preceding principles. (8) A knowledge of the correct terminology of all subjects covered, this knowledge to be obtained through rational use of the ideas.

EIGHTH GRADE.

It is expected that the eighth grade will accomplish the following results:

(1) An adequate and final knowledge of the necessity and meaning of negative numbers and their relation to positive numbers; of the four fundamental operations as applied to the realm of positive and negative number; of the meaning and methods of transformation of simple equations in one

and two unknowns; of the graphical representation of simple equations in two unknowns. (2) The geometrical work consists in the demonstration of the properties of plane figures by induction and in the method of establishing equality by superposition, in proving the simpler principles of similarity, and in the exemplification of algebraic work. (3) A knowledge of the elements of descriptive geometry, so far as it relates to the construction of the plans and elevations of the simpler geometrical and plane sections of them. (4) A study of the geometry of the propositions of the "triangle of forces," of the "parallelogram of forces," of the "polygon of forces," of the laws of equilibrium, and other elementary questions of mechanics which depend for their proofs on simple geometrical considerations. (5) Great care is to be exercised that new pupils who are weak in their arithmetic shall make up these delinquencies before spending much time on these more advanced questions. (6) Occasionally the experimental bases of some problems will be supplied, but only so far as limited time will warrant. It is not the intention to make this work a course in elementary experimental physics, nor shall the thinking demanded by the mathematics of the problem be too seriously diluted with manipulation.

While it will not be possible for us to accomplish fully all the aims enumerated above with the new pupils, whose mathematical preparation is so varied and so poor as is at present the case, we must keep them in mind as the goal to be reached when pupils reach our work through our own channels. Furthermore, nothing has been said as to how the teacher shall present this work to her class, save that material shall be drawn at all times as largely as possible from the other subjects. Caution must here be given, however, against undertaking, during the short period assigned to number, to do all the experimental work from which the data of problems are to be derived. The number classes should not even be required to do very much of their collating of materials *on number time.* The data are to be drawn from the other subjects—to be gathered during the periods assigned to these subjects—and *merely discussed and reduced* in the number class. Any other means of collecting data may result in the teaching of physics or chemistry or nature study; but let no one deceive herself with the idea that she is securing mathematical results of any consequence. Naming a subject number because number is involved in it in a more or less indefinite and uncertain way is to caricature whatever virtue there may be in the idea of correlation.

These outlines are to be understood as merely setting in mathematical terms the goal to be reached from grade to grade in what is believed to be a rational and perfectly feasible course, leading to and preparing for the secondary school in such a way as to meet practical requirements of everyday life much better than does the usual curriculum, and to make the elementary and secondary schools organic parts of a homogeneous whole. It would be both unwise and impracticable to give to each teacher specific directions as to *how* each step is to be taken. To think out means for her particular grade is both the privilege and the duty of each teacher. What the subject-matter of mathematics is, and how it shall be presented from day to day, depends so largely upon what the other work is that the teacher who plans the other work is, in fact, the only one who can successfully deal with the question of the way the ends specified above are to be realized in her own grade. Successful correlation can accomplish the specified results, and to be successful correlation cannot be satisfied with less.

NOTE.—The work in applied mathematics, outlined last month, has been delayed by cloudy weather and other circumstances. It will therefore be finished during this month.

NATURAL SCIENCE.

WILBUR S. JACKMAN.

REVIEW OF WORK FOR OCTOBER.

The study has been based almost wholly upon the subject-matter furnished by outdoor observation.

1. A field trip to North Chicago gave opportunity to observe the development of a valley through erosion. This valley furnished fine illustrations of how plants colonize and adapt themselves to the varied conditions presented along the course of the stream.

The head of the stream is surrounded by a dry upland covered with fine grass. In one part of this there is a slight depression, which, though dry at the time of the visit, being covered with sedges and punctured with holes made by crawfish, bespoke the nearness of water. A few yards below, in the direction indicated by a shallow trough in the surface, running water appeared. A small pool that was formed was almost clogged with algæ and fine rushes. As the stream increased somewhat in size and by its winding formed miniature marshes, sedges, rushes, flags, cat-tails, and willows became more and